ERRATUM

A. Höring pointed out that [DP13, Lemma 12] is wrong as stated. Indeed, let Y be a general quartic surface in $\mathbb{P}^3$ and set $X := \mathbb{P}_Y(\Omega^1_Y)$ with tautological divisor L and natural morphism $\pi\colon X \to Y$. Let H be the pull-back of a hyperplane section on Y. Then $D := L + 2H$ is an ample divisor on X by [GO20, Proposition 3.1] while $D^2 \cdot L < 0$. The conclusion of [DP13, Lemma 12] fails for a general complete intersection curve C of elements of $|mD|$ ($m \gg 1$) since there is a surjective morphism $\pi^*\Omega^1_Y \to \mathscr{O}_X(L)$ so that $(\pi^*\Omega^1_Y)_{|C}$ is not nef.

It is not true in general, as claimed at the bottom of page 590 of [DP13], that the maximally destabilizing subsheaf is invariant under the action of the Galois group G since the polarization is not invariant under G. As a consequence, [DP13, Corollary 13] is also wrong as stated.

To correct the proof of Theorem 18 of [DP13] which relies on [DP13, Corollary 13], one can argue as follows.

Lemma. *Let X be a smooth complex projective variety and let X° be an open subset of X with $\operatorname{codim}_X(X \setminus X^\circ) \geq 2$. Let Y° be a smooth variety with $\dim(Y^\circ) \geqslant 1$ and let $\pi^\circ\colon X^\circ \to Y^\circ$ be a proper surjective equidimensional morphism with reduced fibers. Suppose that a general fiber F of π° is a Fano manifold with $\rho(F) = 1$. Let $\overline{Y}$ be a normal projective birational model of Y°. Suppose that $\overline{Y}$ is not uniruled. Then there exists a complete curve $C \subseteq Y^\circ$ passing through a general point such that $\Omega^1_{Y_0|C}$ is nef.*

Proof. Since K_X is not pseudo-effective by assumption, we may run a minimal model program for X and end with a Mori fiber space (see [BCHM10, Corollary 1.3.3]). Therefore, there exists a sequence of maps

$$\begin{array}{ccccccccccccc} X := X_0 & \overset{\psi_0}{\dashrightarrow} & X_1 & \overset{\psi_1}{\dashrightarrow} & \cdots & \overset{\psi_{i-1}}{\dashrightarrow} & X_i & \overset{\psi_i}{\dashrightarrow} & X_{i+1} & \overset{\psi_{i+1}}{\dashrightarrow} & \cdots & \overset{\psi_{m-1}}{\dashrightarrow} & X_m \\ &&&&&&&&&&&& \big\downarrow{\scriptstyle \pi_m} \\ &&&&&&&&&&&& Y_m \end{array}$$

where the ψ_i are either divisorial contractions or flips, and π_m is a Mori fiber space. The spaces X_i are normal, $\mathbb{Q}$-factorial, and X_i has terminal singularities for all $0 \leqslant i \leqslant m$. Moreover, $\operatorname{Exc}\psi_i$ is covered extremal rational curves. Since $\overline{Y}$ is not uniruled and $\rho(F) = 1$, we see that general fibers of π° must be disjoint from $\operatorname{Exc}\psi_i \circ \cdots \circ \psi_0$ for all i.

Let Z be a resolution of the graph of $\psi := \psi_{m-1} \circ \cdots \circ \psi_0$ with natural morphisms $p\colon Z \to X$ and $q\colon Z \to X_m$. Shrinking Y° and using the miracle flatness theorem (see [Mat89, Theorem 23.1]), we may assume without loss of generality that $Z^\circ \to Y^\circ$ is flat where $Z^\circ := p^{-1}(X^\circ)$. By the rigidity lemma (see [MFK94, Proposition 6.1]), there exists a morphism $\iota\colon Y^\circ \to Y_m$ and a commutative diagram as follows:

$$\begin{array}{ccc} X^\circ & \overset{\psi|_{X^\circ}}{\dashrightarrow} & X_m \\ {\scriptstyle \pi^\circ}\big\downarrow & & \big\downarrow{\scriptstyle \pi_m} \\ Y^\circ & \underset{\iota}{\longrightarrow} & Y_m. \end{array}$$

The rational map ψ induces an isomorphism from an open set $X^{\circ\circ} \subseteq X^\circ$ onto an open set $X_m^{\circ\circ}$ contained in the smooth locus of X_m with $\operatorname{codim}_{X_m}(X_m \setminus X_m^{\circ\circ}) \geq 2$. Moreover, we may assume that the general fiber $F \subset X^{\circ\circ}$. Let $Y_m^\circ \subseteq Y_m$ be an open set with $\operatorname{codim}_{Y_m}(Y_m \setminus Y_m^\circ) \geq 2$ contained in the smooth locus of Y_m such that the induced morphism $X_m^\circ := \pi_m^{-1}(Y_m^\circ) \to Y_m^\circ$ is equidimensional. Replacing $X_m^{\circ\circ}$ by $X_m^{\circ\circ} \cap X_m^\circ$, we may assume that $X_m^{\circ\circ} \subseteq X_m^\circ$.

Notice that ι is birational. It follows that Y_m is not uniruled. Let $B \subseteq Y_m^\circ \subseteq Y_m$ be a general complete intersection curve in the sense of Mehta-Ramanathan for $(\Omega^1_{Y_m})^{**}$. Arguing as in the last paragraph of the proof of [DP13, Lemma 12] and using the fact that Y_m is not uniruled, we see that $\Omega^1_{Y_m|B}$ is nef. Let

$B_1 \subseteq \pi_m^{-1}(B)$ be a general complete intersection curve. By general choice of B_1 and using the fact that $\operatorname{codim}_{X_m}(X_m \setminus X_m^{\circ\circ}) \geq 2$, we have $B_1 \subseteq X_m^{\circ\circ} \cong X^{\circ\circ}$. Observe that the restrictions of π° and $\pi_m \circ (\psi_{|X^{\circ\circ}})$ to $X^{\circ\circ} \cong X_m^{\circ\circ}$ induce the same fibration on $X^{\circ\circ}$ since $\pi_m \circ (\psi_{|X^{\circ\circ}}) = \iota \circ (\pi^\circ{}_{|X^{\circ\circ}})$ and since both $\pi^\circ{}_{|X^{\circ\circ}}$ and $\pi_m \circ (\psi_{|X^{\circ\circ}})$ are equidimensional with fibers of the same dimension. This in turn implies that π_m has reduced fibers along B_1. Set $B_2 := \psi^{-1}_{|X^{\circ\circ}}(B_1) \cong B_1$. By general choice of B_1, we may therefore also assume that π_m (resp. π°) has smooth fibers along B_1 (resp. B_2). Then

$$((\pi^\circ)^*\Omega^1_{Y^\circ})_{|B_2} \cong (T_{X^\circ}/T_{X^\circ/Y^\circ})^*_{|B_2} \cong (T_{X_m}/T_{X_m/Y_m})^*_{|B_1} \cong ((\pi_m)^*\Omega_{Y_m^1})_{|B_1}.$$

In particular, if $C := \pi^\circ(B_2)$, then $\Omega^1_{Y_0|C}$ is nef. □

The proof of Corollary 13 in [DP13] shows that the following holds.

Lemma. *Let X be a smooth complex projective variety and let X° be an open subset of X with* $\operatorname{codim}_X(X \setminus X^\circ) \geq 2$. *Let Y° be a smooth variety and let $\pi^\circ \colon X^\circ \to Y^\circ$ be a proper surjective morphism. Assume that the generic fiber of π° is isomorphic to a projective space. Let $\overline{Y}$ be a normal projective birational model of Y°. If $\overline{Y}$ is uniruled, then there exists a minimal free morphism $f : \mathbf{P}^1 \to Y_0$.*

The proof of Theorem 18 can then be easily adapted.

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