ERRATUM

Adrian Langer pointed out that the proof of [Dru18, Proposition 5.4] is incomplete. It is not true in general that a general member of a base-point-free linear system is smooth. As a result [Dru18, Proposition 5.4] as stated is unproven. That proposition should read as follows.

Proposition. *Let X be a smooth projective variety over an algebraically closed field k of positive characteristic p, and let H be a big semiample divisor on X. Suppose that there exists a smooth and connected curve that is a complete intersection of members of $|mH|$ for some sufficiently large integer m. Suppose in addition that T_X is H-semistable and that $\mu_H(T_X) \geqslant 0$. Let $\mathscr{E}$ be a coherent locally free sheaf on X. Suppose furthermore that $p \geqslant \operatorname{rank} \mathscr{E} + \dim X$. If $\mathscr{E}$ is H-semistable, then so is $\mathrm{F}^*_{\mathrm{abs}}\mathscr{E}$.*

The proof of [Dru18, Theorem 1.4] can then be easily adapted as the situation comes from reduction from characteristic zero.

The proof of [Dru18, Corollary 8.3] is also incomplete. That statement should read as follows.

Proposition. *Let Y and Z be complex projective varieties, and let Z° be an open subset such that $Y^\circ := Y \cap Z^\circ \subseteq Y_{\mathrm{reg}}$ and such that $Y \setminus Y^\circ$ has codimension $\geqslant 2$. Let H be an ample Cartier divisor on Z. Let $V \subset Z^\circ$ be a germ of smooth locally closed analytic submanifold along Y° in Z. Then V is algebraic if and only if there exists $c > 0$ such that, for any positive integer j and any section $s \in H^0\big(Z, \mathscr{O}_Z(jH)\big)$ such that $s_{|V}$ is non-zero, the multiplicity $\mathrm{mult}_{Y^\circ}(s_{|V})$ of $s_{|V}$ along Y° is $\leqslant cj$.*

Proof. The proof is similar to that of [Bos04, Proposition 2.2], [Bos01, Corollary 3.8] or [CP19, Theorem 4.2], and so we leave some easy details to the reader.

Let X be the Zariski closure of V and set $d := \dim V$. We have to show that there exists $C > 0$ such that $H^0\big(X, \mathscr{O}_X(jH_{|X})\big) \leqslant Cj^d$ for any $j \gg 1$.

Given $i \geqslant 0$, let $V_i \subset V$ be the scheme defined by $\mathscr{I}^{i+1}_{Y^\circ/V}$, and let $F^iH^0\big(X, \mathscr{O}_X(jH_X)\big) := \big\{s \in H^0\big(X, \mathscr{O}_X(jH_{|X}\big) \,|\, s_{|V_i} \equiv 0\big\}$. By our assumption, $F^iH^0\big(X, \mathscr{O}_X(jH_X)\big) = 0$ if $i > cj \gg 1$. Set $X^\circ := X \setminus \big(Y \setminus Y^\circ\big)$. Note that $X \setminus X^\circ$ has codimension at least two.

Then

$$\begin{aligned} h^0\big(X, \mathscr{O}_X(jH_{|X})\big) &\leqslant \sum_{0 \leqslant i \leqslant cj} \dim F^iH^0\big(X, \mathscr{O}_X(jH_X)\big)/F^{i+1}H^0\big(X, \mathscr{O}_X(jH_X)\big) \\ &= \sum_{0 \leqslant i \leqslant cj} \dim F^iH^0\big(X^\circ, \mathscr{O}_{X^\circ}(jH_{X^\circ})\big)/F^{i+1}H^0\big(X^\circ, \mathscr{O}_{X^\circ}(jH_{X^\circ})\big) \\ &\leqslant \sum_{0 \leqslant i \leqslant cj} h^0\big(Y^\circ, S^i\, \mathscr{N}^*_{Y^\circ/V} \otimes \mathscr{O}_{Y^\circ}(jH_{Y^\circ})\big) \\ &= \sum_{0 \leqslant i \leqslant cj} h^0\big(Y, S^{[i]}\, \mathscr{C} \otimes \mathscr{O}_Y(jH_Y)\big), \end{aligned}$$

where $\mathscr{C}$ denotes the reflexive extension of $\mathscr{N}^*_{Y^\circ/V}$ to Y. Note that $\mathscr{C}$ has rank $d - \dim Y + 1$.

A construction due to Nakayama (see [Nak04, pp. 202-203]) together with the asymptotic Riemann-Roch formula then show that there exists $C > 0$ such that $h^0\big(Y, S^{[i]}\, \mathscr{C} \otimes \mathscr{O}_Y(jH_Y)\big) \leqslant C(i+j)^{d-1}$. Our claim then follows easily. □

The proof of [Dru18, Proposition 8.4] remains unchanged.

References

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